Session – 01

Fluid Statics

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Density

It is defined as the mass per unit volume of a substance.

$$\rho = \frac{M}{V}$$

Density of water is
1000 kg m^{-3}
or
1 g cm^{-3}

- Density is a scalar quantity
- SI unit of density is kg m^{-3}
- Dimensional formula of density is $[\, M^1 L^{-3} \,]$

Relative density

Relative density of a substance is defined as the ratio of density of the substance to density of water.

$$RD = \frac{\rho_{\text{material}}}{\rho_{\text{water}}}$$

Relative density may be greater than, equal to or lesser than 1

- Relative density is a scalar quantity
- It has no units or dimensions

Pressure

Pressure is defined as the normal force per unit area.

$$\boxed{P = \frac{F}{A}}$$

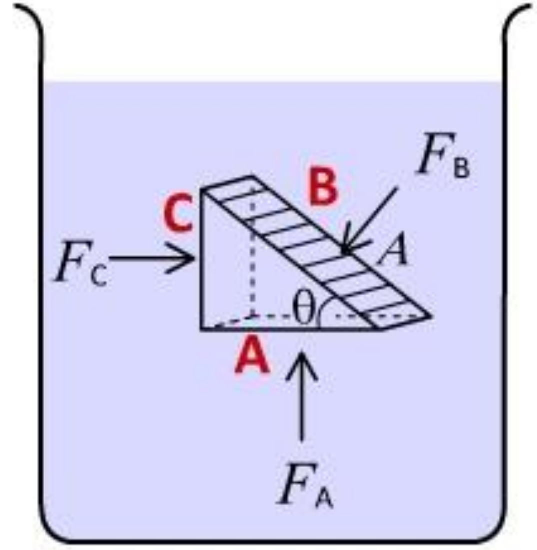
- Pressure is (treated as) a scalar quantity (as it is due to only the *normal component* of the force)
- SI unit of pressure is Nm^{-2} or pascal
- Other units of pressure are torr, bar, PSI etc.
- Dimensional formula of pressure is $[\ ML^{-1}T^{-2}\ ]$
- Atmospheric pressure (at sea level) is 1.01×10^5 pascals

<u>Absolute pressure</u> : is the total pressure exerted at a point.

<u>Gauge pressure</u> : Difference between absolute pressure and atmospheric pressure.

Pascal's law

Pressure in a fluid at rest is equal in all directions.



Consider liquid in a stationary container. Consider a small segment of the liquid, in the shape of a triangular prism. Let A, B & C be the three faces and let F_A, F_B and F_C be the forces acting on the segments respectively (as shown in the figure) . The liquid is in equilibrium.

Force F_B may be resolved into horizontal and vertical components as

$$F_{By} = F_B \cos(\theta) \qquad F_{Bx} = F_B \sin(\theta)$$

Considering the equilibrium of the segment

$$F_C = F_B \sin(\theta) \quad \text{— i}$$

$$F_A = F_B \cos(\theta) \quad \text{— ii}$$

Considering the geometry

$$A_C = A_B \sin(\theta) \quad \text{— iii}$$

$$A_A = A_B \cos(\theta) \quad \text{— iv}$$

Dividing eq(i) by eq (iii) and eq(ii) by eq(iv) we get

$$\boxed{\frac{F_A}{A_A} = \frac{F_B}{A_B} = \frac{F_C}{A_C}}$$

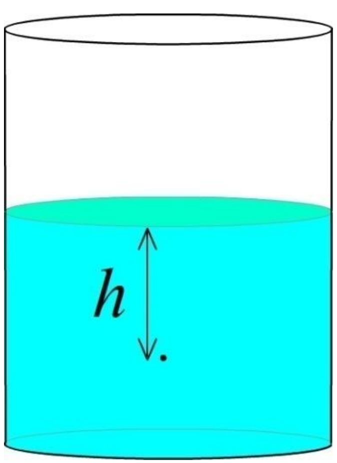
Variation of pressure with depth

Consider a liquid column of height. Let density of the liquid be ρ. Consider a point P at a depth of h from the surface of the liquid.

Force exerted by the liquid column at the surface containing the point P is

$$F = mg$$

$$F = \rho \times \text{vol} \times g$$



$$F = \rho A h g$$

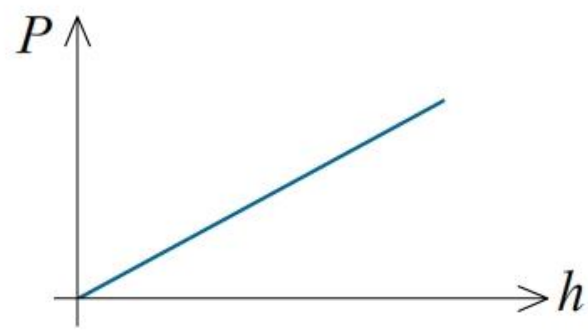
Pressure is given by $P = \frac{F}{A}$

substituting F from above equation

$$P = \frac{\rho A h g}{A}$$

$$\boxed{P = \rho g h}$$

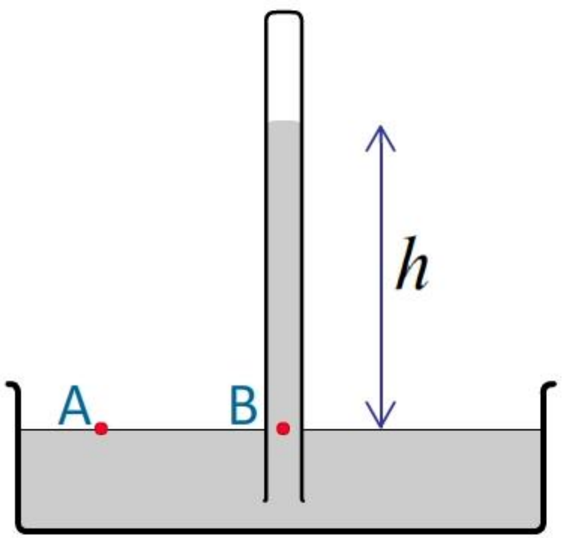
From the above relation it is observed that pressure exerted by a liquid column increases with depth.



Pressure measuring devices

(Measurement of atmospheric pressure : Torricelli's barometer)

A long glass tube closed at one end and filled with mercury is inverted into a trough of mercury as shown in the figure. It is observed that mercury level falls to a height h in the tube and then attains equilibrium. Space above the mercury column in the tube contains only mercury vapour whose pressure is negligibly small.



Under equilibrium, pressure at a point B in the tube is equal to the pressure at point A on the open surface of the liquid. Therefore

$$P_A = P_B$$

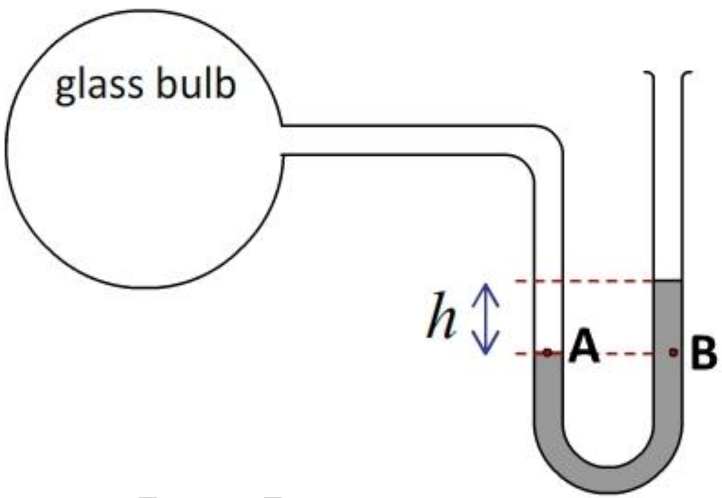
$$P_o = \rho g h$$

Atmospheric pressure is equal to the pressure exerted by a mercury column of height 76 cm.

$\boxed{1\,\text{atm} = 76\ \text{cm of Hg}}$

Pressure measuring devices (Mercury manometer)

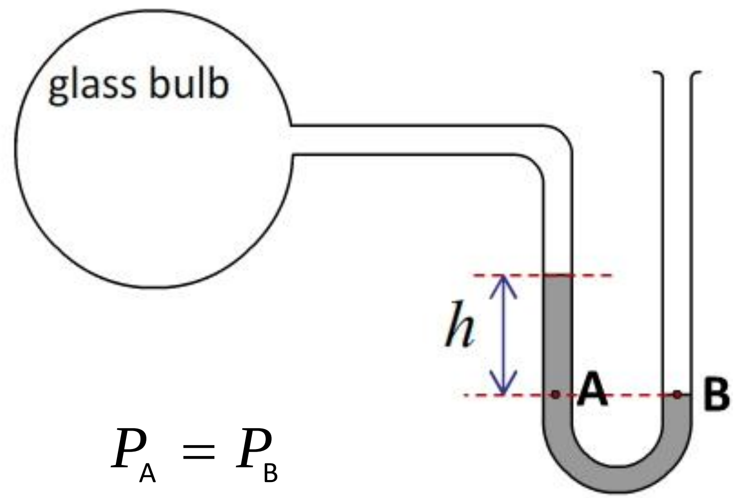
An open tube manometer is a useful instrument for measuring pressure differences. It consists of a U-tube containing a suitable liquid (low density liquid for measuring small pressure differences and a high density liquid for large pressure differences). One end of the tube is open to the atmosphere and the other end is connected to the system whose pressure is to be determined



$$P_A = P_B$$

$$P_{gas} = P_o + \rho_{Hg} g h$$

$$P_{gas} > P_{atmosphere}$$



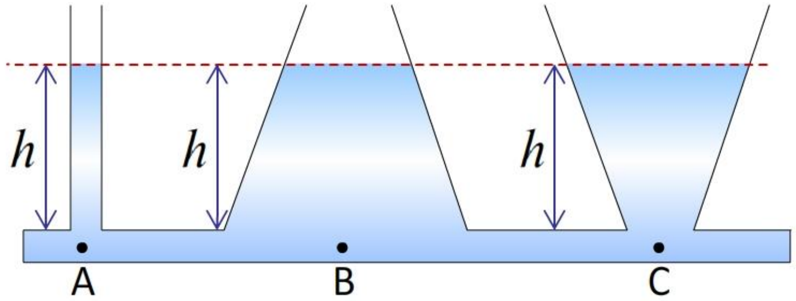
$$P_A = P_B$$

$$P_{gas} + \rho_{Hg} g h = P_O$$

$$P_{gas} = P_O - \rho_{Hg} g h$$

$$P_{gas} < P_{atmosphere}$$

Hydrostatic paradox



Pressure at the base of a container, due to the liquid contained in it, is independent of the shape of the container and depends only on the height of the liquid column.

Pressure is same at the points A, B and C (and it does not depend on the shape of the container)

Hydraulic lift

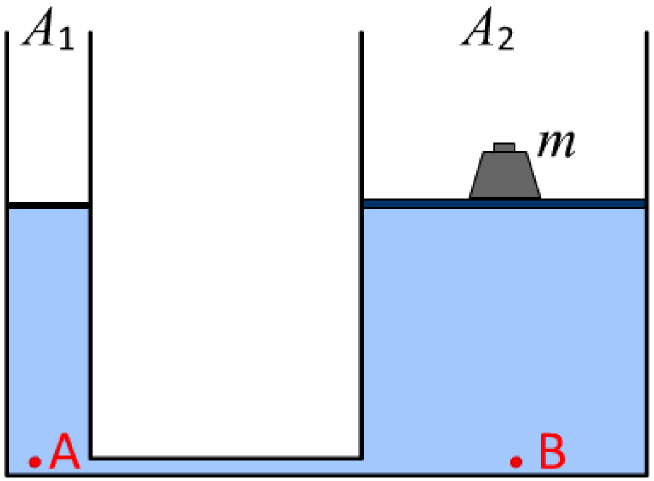
Principle : Pascal's law

Consider two cylindrical containers of areas of cross sections A_1 and A_2, connected to each other. The containers are fitted with pistons assumed to be of negligible mass and friction. Let a load of mass m be placed on the right hand side cylinder.

Under equilibrium, pressure is same at two points A and B which are at the same height. Therefore

$$P_A = P_B$$

$$P_o + \frac{F}{A_1} + \rho g h = P_o + \frac{mg}{A_2} + \rho g h$$



$$\frac{F}{A_1} = \frac{mg}{A_2}$$

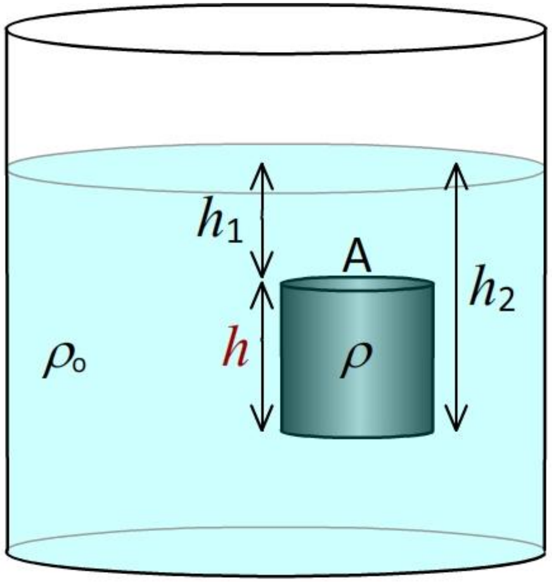
$$F = \frac{A_1}{A_2} \times mg$$

As $A_1 < A_2$, lesser force (F) is required to support the load (mg).

A_2 / A_1 is called the mechanical advantage of the hydraulic lift

Buoyant force

Consider a cylindrical body of area of cross-section A, height h and density ρ placed in a fluid of density ρ_o.



Downward force exerted on it due pressure on the upper surface is

$$F_1 = P_1 A$$

$$F_1 = \rho_o g h_1 A \quad \text{(i)}$$

Upward force exerted on it due pressure on the lower surface is

$$F_2 = P_2 A$$

$$F_2 = \rho_o g h_2 A \quad \text{(ii)}$$

Net upward force exerted on it is

$$F_{net} = \rho_o g A (h_2 - h_1)$$

$$F_{net} = \rho_o A h g$$

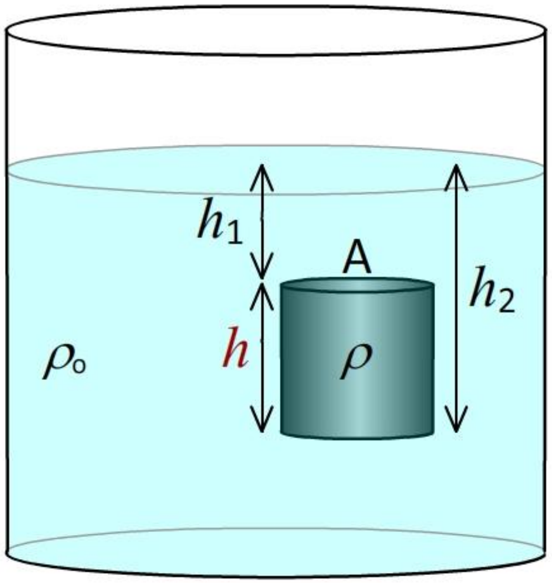
$$F_{net} = \rho_o V g$$

$$F_{net} = m_{liq} g$$

$$F_B = \text{weight of liquid displaced}$$

Buoyant force

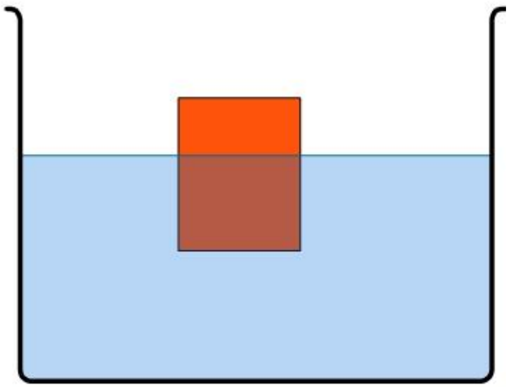
$$F_B = \text{weight of fluid displaced}$$



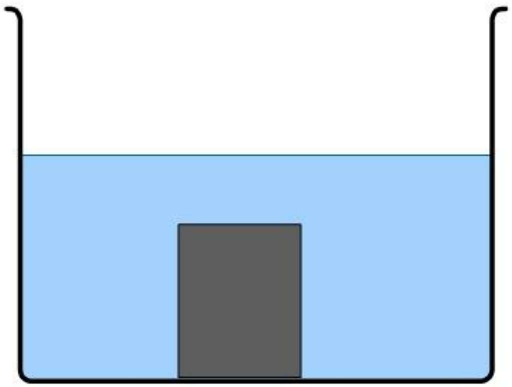
- Line of action of buoyant force passes through the centre of gravity of the object
- Buoyant force acts on an object placed in any fluid
- Buoyant force acts on the body even when the body is stationary in the fluid.
- Buoyant force depends on the volume of the body in the fluid and the density of the fluid
- Buoyant force doesn't depend on the density of the body

Relative density and floatation

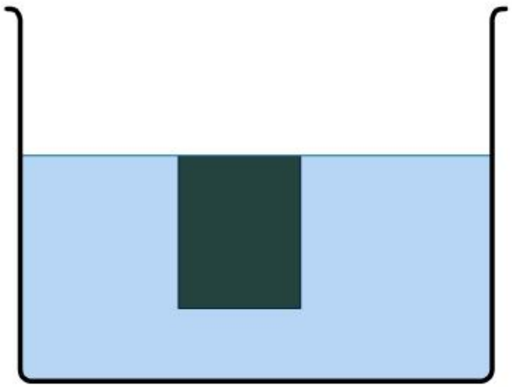
$$RD = \frac{\rho_{body}}{\rho_{water}}$$



Body floats in water
RD < 1



Body sinks in water
RD > 1



Body 'just' floats
RD = 1

Relative density (of bodies that sink in water)

Weight of the body in air is

$$W_1 = mg$$

$$W_1 = \rho V g \quad \text{(i)}$$

When the body is in water then

$$W_2 + F_B = mg$$

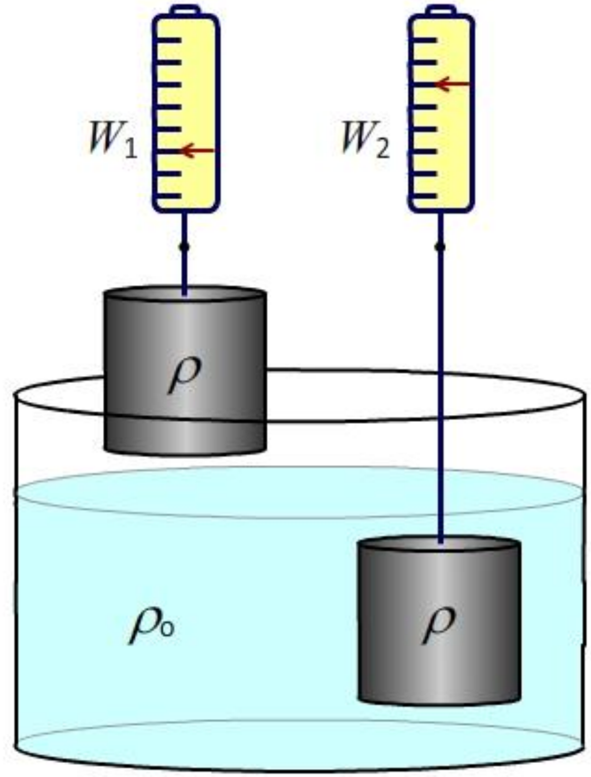
$$W_2 + m_{liq} g = mg$$

$$W_2 + \rho_o V g = W_1$$

$$\rho_o V g = W_1 - W_2 \quad \text{(ii)}$$

Dividing eq(i) by eq(ii)

$$\boxed{\text{RD} = \frac{\rho}{\rho_o} = \frac{W_1}{W_1 - W_2}}$$



W_1 : Weight of body in air

W_2 : Weight of body in water

W_1 - W_2 : Loss of weight in water

W_1 - W_2 : Buoyant force

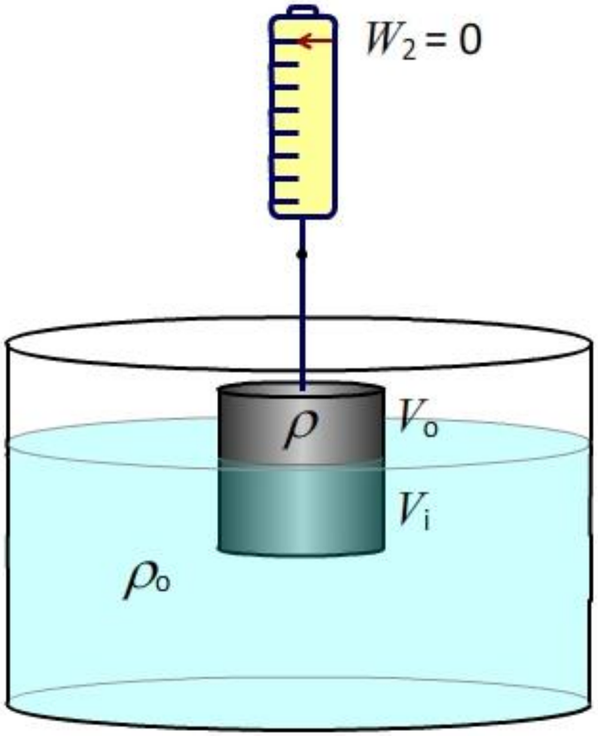
W_1 - W_2 : Volume of solid (in CGS units)

Relative density (of bodies that float in water)

$$\mathrm{RD} = \frac{\rho}{\rho_o} = \frac{V_{\text{immersed}}}{V_{\text{total}}}$$

W_2 : zero

Buoyant force is equal to weight of body in air



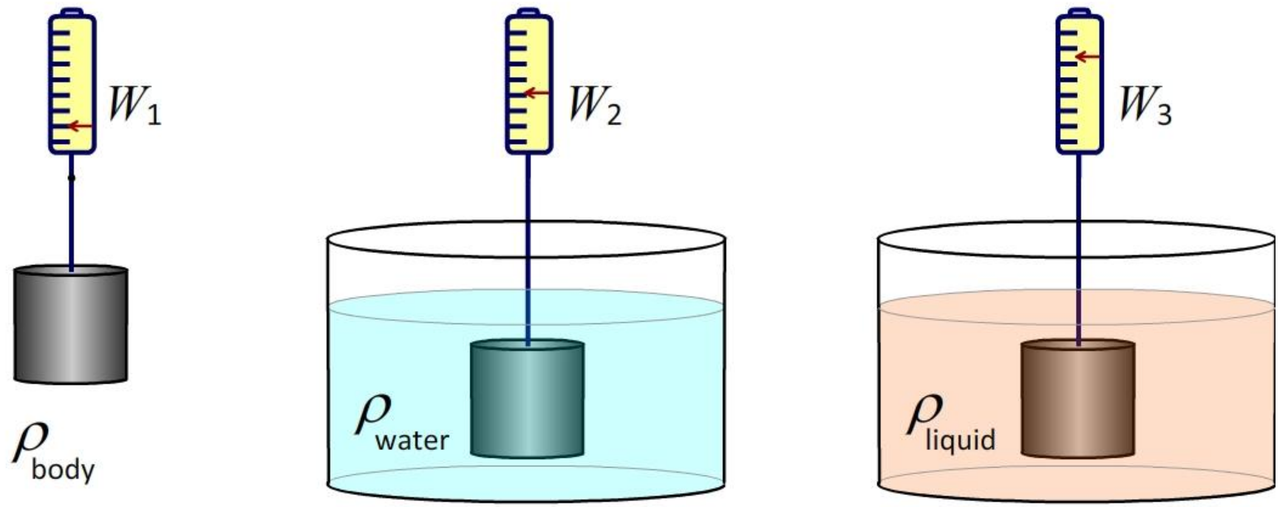
Relative density (of liquids)

$$\mathrm{RD}_{\text{liquid}} = \frac{\rho_{\text{liq}}}{\rho_{\text{o}}} = \frac{W_1 - W_3}{W_1 - W_2}$$

W_1 : Weight of body in air

W_2 : Weight of body in water

W_3 : Weight of body in the liquid



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